

Lecture 12

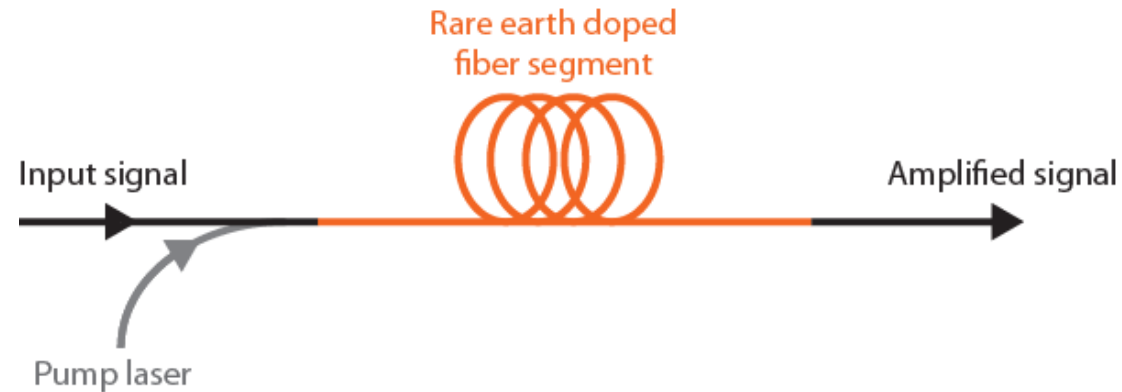
Study of optical amplification

EE 440 – Photonic systems and technology
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Rare earth doped fiber amplifiers

Rare earth doped fiber amplifiers

An important class of amplifiers make use of fibers doped with **rare earth elements** as a gain medium



Optically pumped system: energy of pump photons are used to excite atoms
Fiber core doped with rare earth element during the fabrication process.

- Doping typically in 100 or 1000's ppm

Amplifier properties determined by dopant, silica fiber is just host medium

- Different rare earth elements can be used depending on the wavelength of operation: erbium, holmium, neodymium, thulium, ytterbium etc.

Erbium doped fiber amplifiers

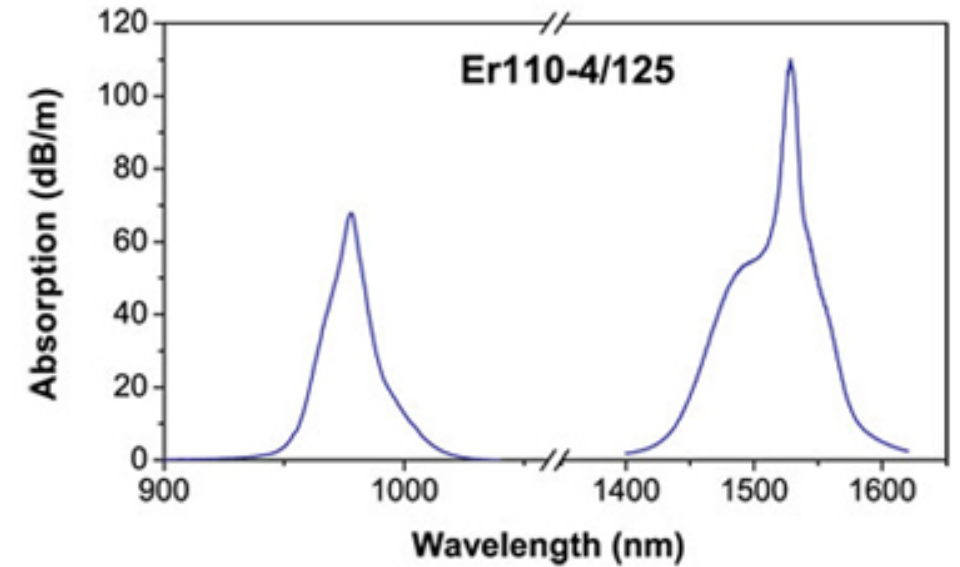
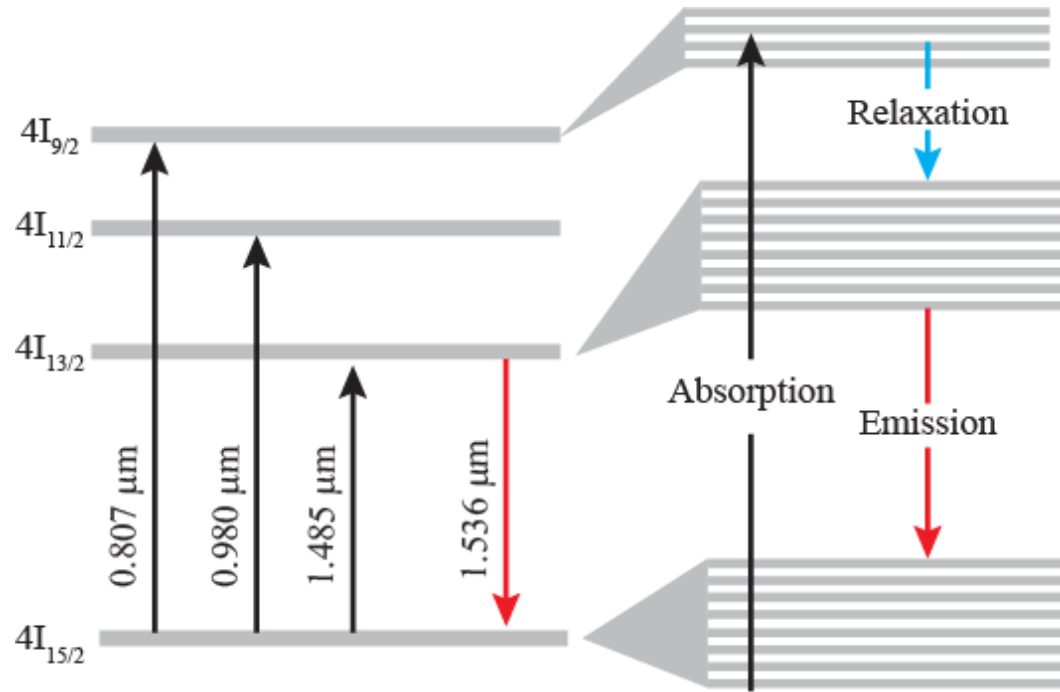
Erbium doped fiber amplifiers have attracted most attention because they operate in the wavelength region of 1550 nm

Fiber core is doped with Erbium Er^{3+}

- Erbium has a transition with very long lifetime at 1550 nm
- Optical pumping at suitable wavelength provides gain through population inversion
- Germanium (Ge), alumina (Al) are also typically added to add solubility of Er and to increase the core index

Gain spectrum depends on pumping scheme as well as on presence of other dopants

Erbium atom energy levels



- Amorphous nature of silica broadens all energy levels into bands (Stark splitting)
- Many different pumping schemes: 980 nm and 1480 nm are favored to avoid excited state absorption (ESA)

EDFA operation

Spontaneous lifetime of $4I_{11/2}$ is about $1\ \mu\text{s}$ while the lifetime of $4I_{13/2}$ is close to $10\ \text{ms}$

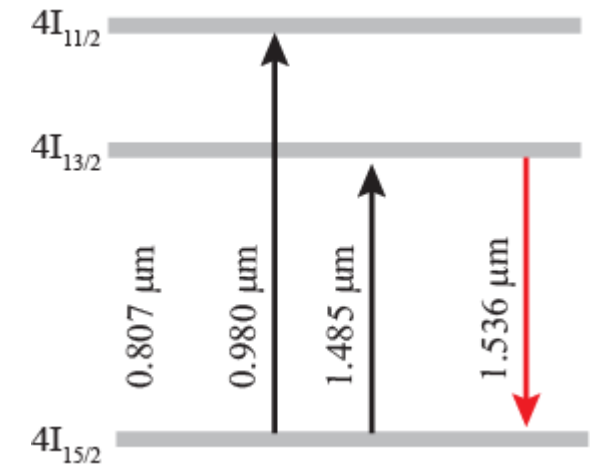
- Fast relaxation to the emission level
- Lifetime of excited state much slower than signal bit rates of practical interest, no distortions

980 nm pumping can achieve high excitation levels

- High gain efficiency
- Low power efficiency

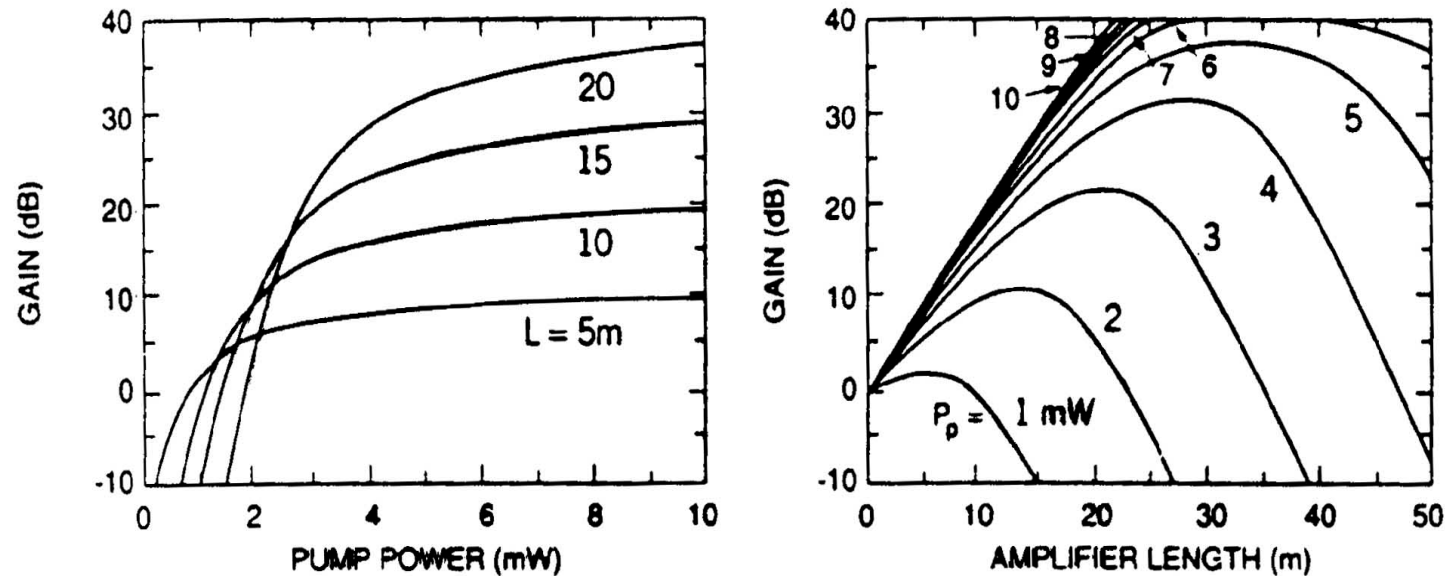
1480 nm pumping directly excited the upper sublevels of the $4I_{13/2}$ metastable state:

- Limits the excitation level due to stimulated emission
- Better power efficiency



EDFA basic: gain

EDFA gain depends on device parameters: Er concentration, fiber length, core radius, pump power, etc.

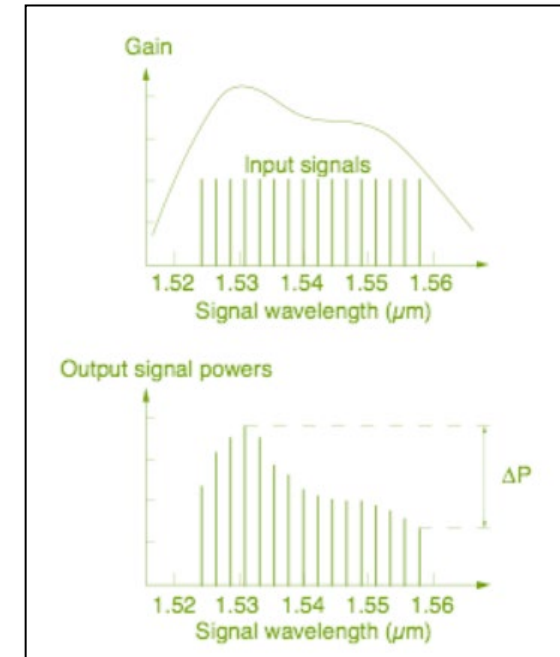
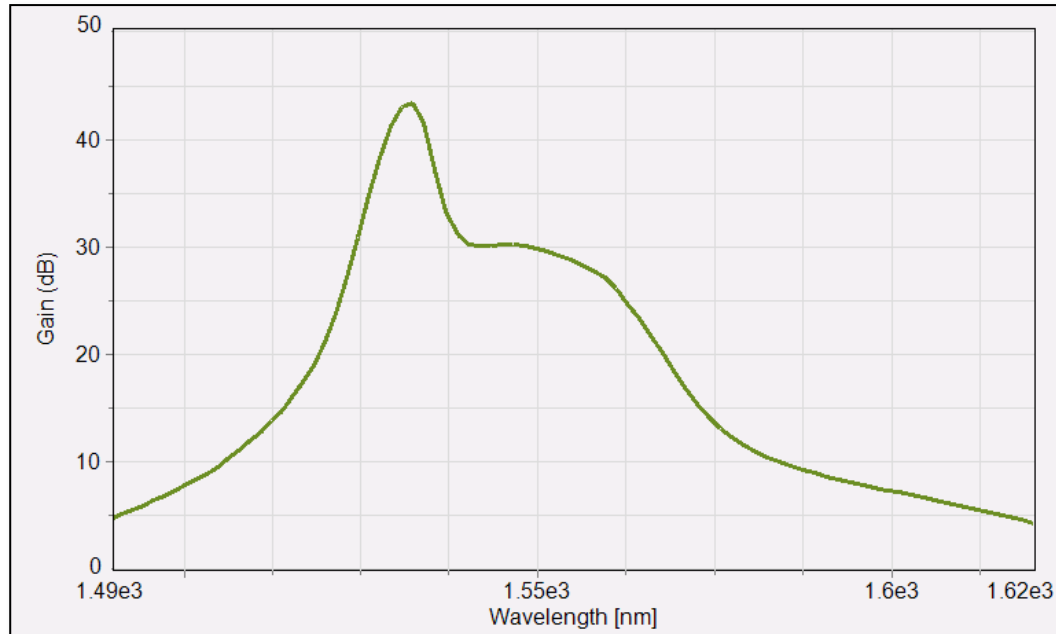


from Agrawal

- For a given amplifier length L , gain initially increases with pump power then saturates
- For a given pump power, amplifier gain reaches a maximum at an optimum length L , then rolls off sharply
- Both L and pump power must be optimized for a particular amplifier design

EDFA basic: gain spectrum

Fiber length of 15 m, 980 nm forward pump with 80 mW.



- Shape of the gain spectrum is affected by the amorphous nature of silica and the presence of other co-dopants within the fiber core
- Broad spectrum (more than 35 nm), double peak structure
- Gain towards L-band (1570-1615 nm) can be obtained with longer fibers

Pumping choices for EDFAs

Pumping direction

- Forward pumping (co-propagation) generates less noise
- Backward pumping generates higher gain

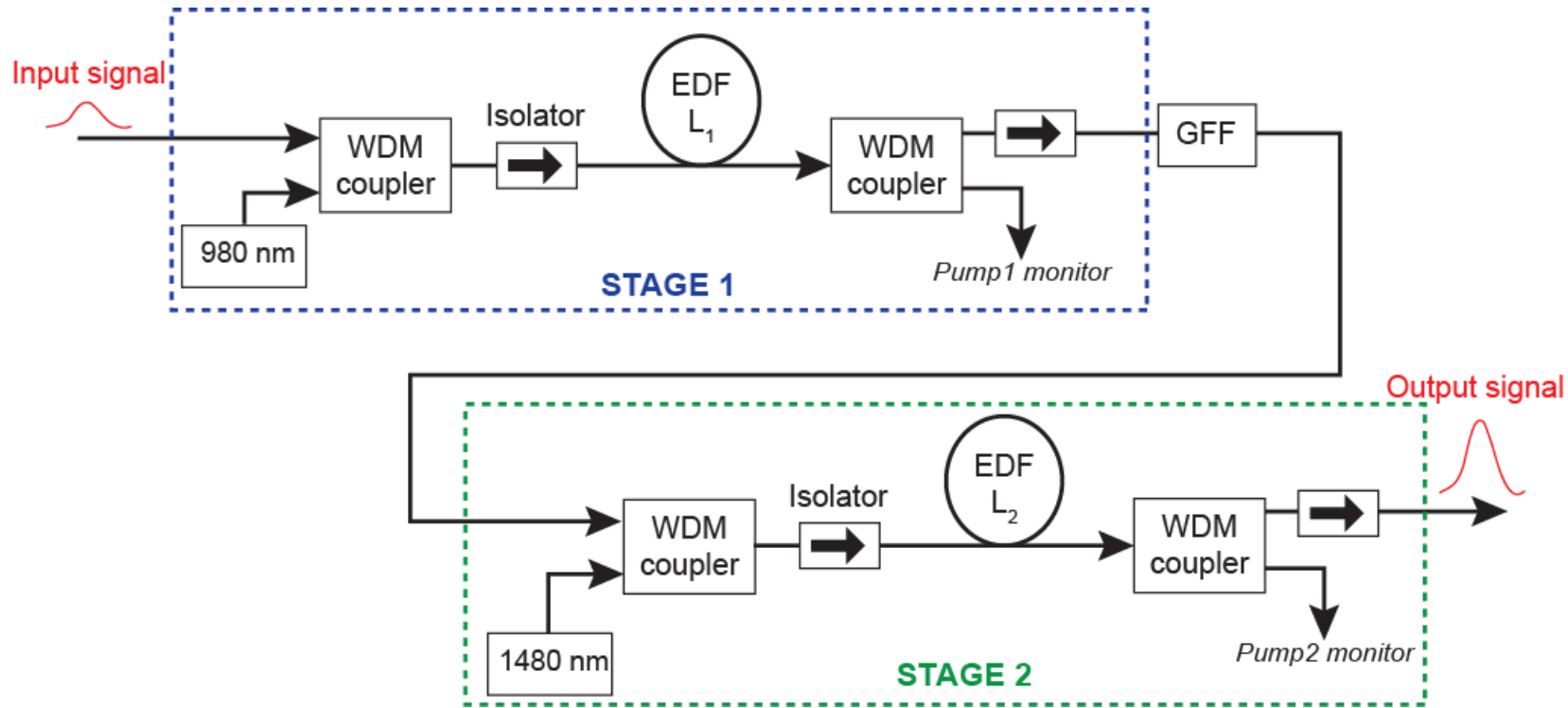
980 nm pumping

- Generates higher gain: can achieve very high excitation levels
- Generates less noise: noise figure is a function of inversion level
- Power efficiency is not ideal

1480 nm pumping

- High power 1480 nm laser diodes available
- Tolerates a broader range of pump wavelengths.
- Limited excitation level and hence gain per unit length
- Flatter gain

Two stage C band EDFA



Stage 1:

- 980 nm pump for high inversion level
- Low noise figure
- Limited output power: preamplifier stage

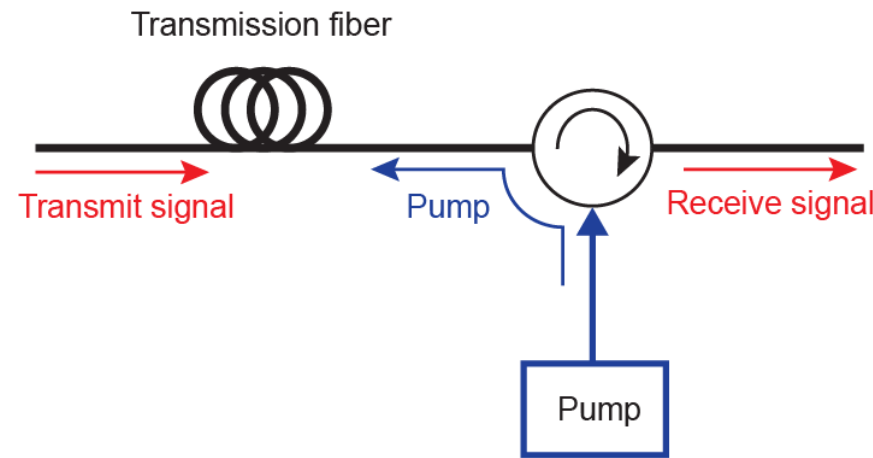
Stage 2:

- 1480 nm pump for high power
- High noise figure
- High output power: booster stage

Raman amplifiers

Raman amplification

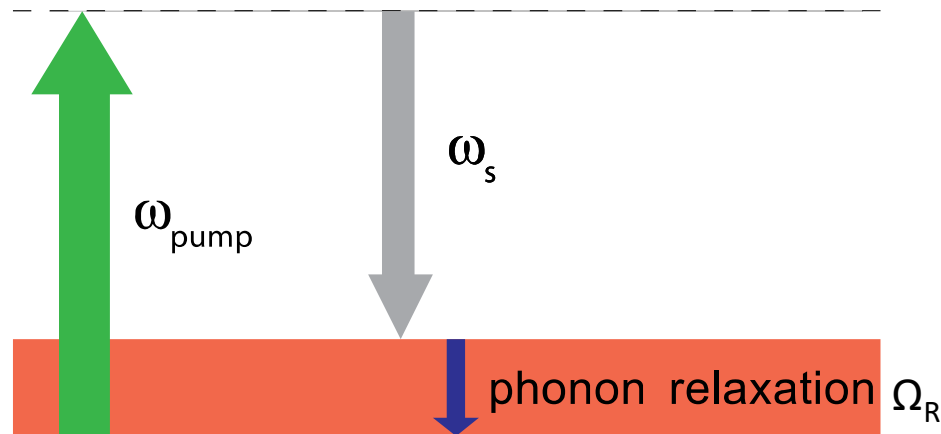
Silica fiber can be used as amplifying medium through **stimulated Raman scattering**: fiber based Raman amplifier



- Optically pumped system, uses intrinsic optical nonlinearities of the silica fiber.
Amplification takes place throughout the length of the transmission fiber.
- Hence also known as distributed amplification.

Spontaneous Raman Scattering

Spontaneous Raman scattering occurs in optical fiber when a pump wave is scattered by the silica molecules

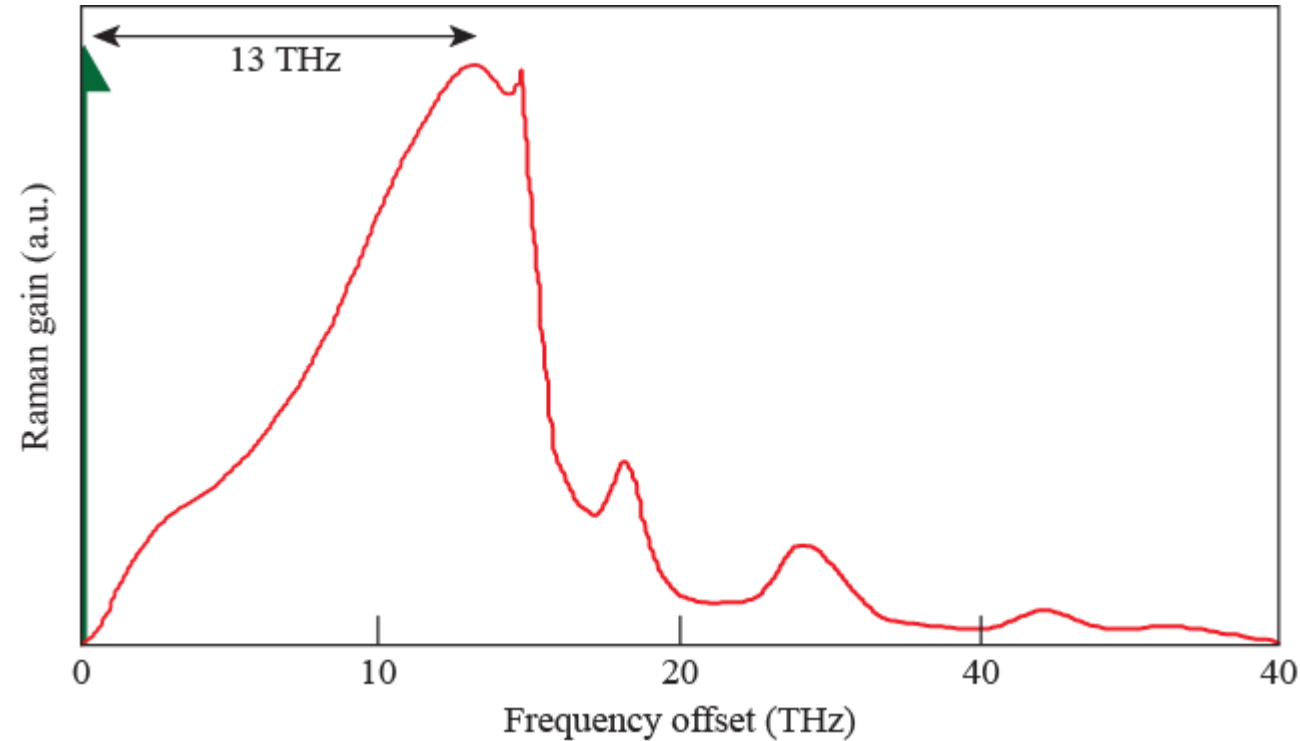


- Photons from pump give energy to photons with reduced energy
- Remaining energy absorbed by silica molecules: vibrational phonons

Energy conservation : $\omega_s = \omega_{\text{pump}} - \Omega_R$

- Dictated by the vibrational energy levels of silica
- Isotropic process, occurs in all direction
- If an input wave at frequency ω_s is present, it will be amplified by the pump

Raman gain spectrum



- Wide gain bandwidth due to amorphous nature of glass
- Maximum gain when Raman shift Ω_R is about 13 THz

Amplifier characteristics

The optical gain coefficient $\gamma(z)$ depends on:

- pump intensity $I_p(z)$
- frequency offset pump-signal through the Raman gain coefficient $g_r(\omega)$

$$\gamma(z) = g_r(\omega)I_p(z)$$

Intensity I_p is related to the pump power P_p

$$I_p(z) = \frac{P_p}{A_{eff}}$$

- A_{eff} can vary significantly for different types of fibers
- DCF can be 8 times more efficient than SMF due to its smaller core diameter

Amplifier characteristics

Variations in pump and signal powers along the amplifier length (i.e. transmission fiber) can be studied through coupled equations

- Signal power evolution:

$$\frac{dP_s}{dz} = -\alpha_s P_s + \frac{g_r}{A_{eff}} P_p P_s$$

P_s, P_p : Signal and pump power
 g_r : Raman gain coefficient
 A_{eff} : Pump cross sectional area
 α_s, α_p : Losses at signal, pump frequencies

- Pump power evolution under small signal amplification (pump depletion is neglected):

$$\frac{dP_p}{dz} \approx -\alpha_p P_p$$

Small signal amplification

$$\frac{dP_p}{dz} \approx -\alpha_p P_p \Rightarrow P_p(z) = P_0 \exp(-\alpha_p z)$$

$$\text{Solve : } \frac{dP_s}{dz} = -\alpha_s P_s + \frac{g_r}{A_{eff}} P_0 \exp(-\alpha_p z) P_s$$

For amplifier of length L , get:

$$P_s(L) = P_s(0) \exp\left(g_r P_0 \frac{L_{eff}}{A_{eff}} - \alpha_s L\right)$$

$$L_{eff} = \frac{1 - \exp(-\alpha_p L)}{\alpha_p}$$

Small signal amplification

In the absence of Raman amplification have:

$$P_s^{off}(L) = P_s(0) \exp(-\alpha_s L)$$

Amplifier gain is thus given by:

$$G_A = \frac{P_s(L)}{P_s^{off}(L)} = \exp\left(g_r P_0 \frac{L_{eff}}{A_{eff}}\right) \equiv \exp(\gamma_0 L)$$

Small signal gain coefficient γ_0 is:

$$\gamma_0 = g_r \frac{P_0}{A_{eff}} \frac{L_{eff}}{L}$$

Gain saturation

Amplification factor initially increases exponentially with P_0

Deviates as P_0 becomes large: gain saturation

- Pump supplies energy for signal amplification: begins to deplete when P_s increases
- In a typical configuration, the amplifier gain is decreased by 3 dB when amplified signal power becomes comparable to the input pump power
- Since typical P_0 are close to 1 W, Raman amplifiers operate most of the time in the unsaturated regime

Raman amplifier design

Pump and signal injected into the fiber through a fiber coupler.

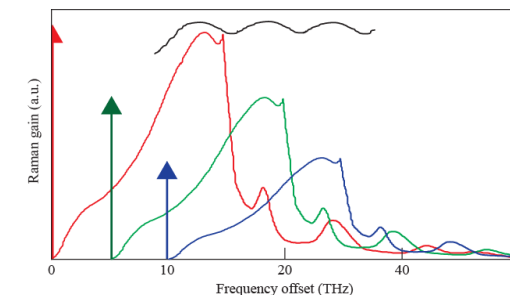
Typical configuration uses backward pumping.

- Raman has a fast response time: enables amplitude fluctuations of the pump to be directly transferred to the signal
- Backward pumping: amplitude fluctuations are averaged out

Optimize frequency of Raman pump with respect to the signal(s)

- Maximum gain is for a frequency offset of 13.2 THz (100 nm near 1550nm)
- Usable gain bandwidth is about 48 nm

Can use multiple pumps to generate broad gain spectrum:



Optically pumped amplifiers

Characteristic	EDFA	Raman
Amplification band	Dopant dependent	Depends on pump wavelengths
Bandwidth	20 nm , more for multiple dopants	48 nm, more for multiple pumps
Gain	30 dB or more, depends on ion concentration, length, pump configuration	4-11 dB proportional to pump intensity and effective fiber length
Saturation power	Depends on gain and material constants	Equals about the pump power
Pumping wavelength	980nm or 1480nm	13.2 THz offset with the signal to amplify

Amplifiers and noise

Amplifier noise

All amplifiers add noise

- Lumped and distributed amplification have different performance

Noise comes from spontaneously emitted photons that have random:

- Direction
- Polarization
- Frequency (within the amplification band)
- Phase

Some will add incoherently to the signal

- Cause intensity and phase noise

Definition of the optical SNR

Optical signals are often characterized by the optical SNR (OSNR):

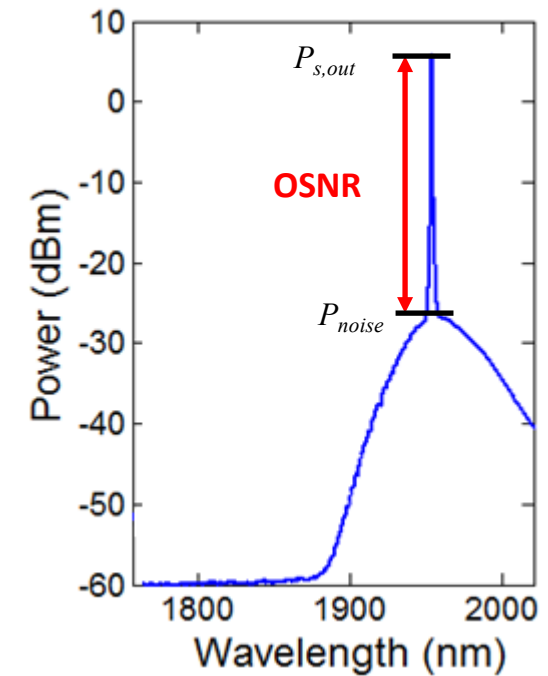
- Easily measured with an optical spectrum analyzer (OSA).
- Very popular as easily monitored in the laboratory.

Definition:

$$OSNR = \frac{P_{signal,X} + P_{signal,Y}}{P_{noise,X} + P_{noise,Y}}$$

Usually the OSNR is normalized to a 0.1 nm bandwidth

- Entire signal is measured, noise measured over 0.1 nm
- Implies that required OSNR for given BER is bit-rate dependent.



EDFA noise

The noise is called amplified spontaneous emission (ASE).

- Due to spontaneous emission of uncorrelated photons.
- They are also amplified if generated within the gain bandwidth of the EDFA.

The output ASE noise power per polarization (x2 for unpolarized signals) is approximately given by:

$$P_{ASE}^{out} = S_{ASE} \Delta\nu_0$$

noise power spectral density

$$S_{ASE} = n_{sp} h\nu (G - 1)$$

spontaneous emission factor
(or population inversion factor)

$$n_{sp} \approx \frac{N_2}{N_2 - N_1}$$

Effective bandwidth of noise set by the
amplifier or optical filter bandwidth

OSNR due to EDFA noise

The OSNR is reduced each time the signal is amplified:

- Each EDFA adds to the noise power spectral density due to the generation of ASE.

Let's assume an amplified link:

- Has N_A amplifiers equally spaced by a given span length
- Span loss is equal to the gain of each EDFA (i.e. link is perfectly loss compensated)
- EDFAs have identical noise performances

$$OSNR_{end} = \frac{P_{in}}{N_A 2P_{ASE}} = \frac{P_{in}}{N_A 2n_{sp} h\nu (G - 1) \Delta\nu_0} \approx \frac{P_{in}}{N_A 2n_{sp} h\nu G \Delta\nu_0}$$

In logarithmic scale for a 1550 nm signal and $\Delta\nu_0 = 0.1$ nm:

$$OSNR_{dB} = P_{in}[dBm] - N_A[dB] - 2n_{sp}[dB] - G[dB] + 58dBm$$

Example

What is the maximum transmission distance with 100 km or 50 km EDFA spacing given that:

- A 10 Gb/s system requires an OSNR of 20 dB
- The loss is 0.25 dB/km and $2n_{sp} = 5\text{dB}$
- The launch power into each span is 1 mW per channel

Electrical signal to noise ratio (SNR)

The Q and BER are linked to the SNR in the detected current

- If signal can be improved we should get an improvement of the SNR.
- How about simply doing optical amplification of the signal before detection?

An EDFA can improve the sensitivity of a thermally limited receiver.

- This is a [pre-amplified optical receiver](#).
- The added optical noise can be much smaller than the thermal noise.

The most important issue in designing optical pre-amplification is therefore the contamination of the amplified signal by the ASE

- The noise can beat with the signal and with itself, generating new frequencies that can fall within the detector bandwidth.

Electrical signal to noise ratio

Received current now has new noise components due to presence of ASE:

- Leads to a signal-ASE beat noise term (i_{sig-sp}) and an ASE-ASE (i_{sp-sp}) beat noise term

$$I = R(GP_{in}) + i_{sig-sp} + i_{sp-sp} + i_s + i_T$$

The variance of the noise terms affected by the ASE are:

$$\sigma_s^2 = 2q[R(GP_{in} + P_{ASE})]\Delta f$$

$$\sigma_{sig-sp}^2 = 4R^2 GP_{in} S_{ASE} \Delta f$$

$$\sigma_{sp-sp}^2 = 4R^2 S_{ASE}^2 \Delta \nu_0 \Delta f$$

Impact of ASE on SNR

Without optical amplification the SNR is given by:

$$SNR_{noAmp} = \frac{(RP_{in})^2}{\sigma_s^2 + \sigma_T^2}$$

After optical amplification and optical bandpass filter, the SNR is:

$$SNR_{Amp} = \frac{(RGP_{in})^2}{\sigma_s^2 + \sigma_T^2 + \sigma_{sp-sp}^2 + \sigma_{sig-sp}^2}$$

Note that:

- Shot noise variance are different in the two cases, thermal remains identical.
- ASE-ASE beating σ_{sp-sp}^2 has typically negligible contribution.
- Noise power contribution P_{ASE} to shot noise has also a negligible contribution.

Impact of ASE

Assume ideal responsivity ($\eta = 1$) and large amplifier gain ($G \gg 1$):

$$R = \frac{q}{h\nu} \text{ and } S_{ASE} = n_{sp} h\nu G$$

Get an expression comparing SNR before and after optical amplification:

$$\frac{SNR_{Amp}}{SNR_{noAmp}} = \frac{1 + k_T}{2n_{sp} + \frac{1}{G} + \frac{k_T}{G^2}} \text{ with } k_T = \frac{\sigma_T^2}{2qRP_{in}\Delta f}$$

- Notice that k_T is the ratio of (thermal noise)/(shot noise) without amplification

The impact of the optical pre-amplification will depend on which noise process dominates before and after amplification.

Optical pre amplification – thermal limit

Example: thermal noise always dominates (i.e. before AND after amplification)

$$\frac{SNR_{Amp}}{SNR_{noAmp}} \approx \frac{(RG P_{in})^2}{\sigma_T^2} \frac{\sigma_T^2}{(R P_{in})^2} \approx G^2$$

- Huge improvement: signal power is increased while noise remain constant.

In realistic case when thermal noise dominates before the optical amplifier, it will be negligible after.

$$\frac{SNR_{Amp}}{SNR_{noAmp}} \approx \frac{k_T}{2n_{sp} + \frac{1}{G}}$$

- SNR improvement saturates with gain but can be very large.

In the thermal limit, optical pre-amplification improves the SNR

Optical pre amplification – shot noise limit

Assume that the optical signal arriving at the receiver already has high power such that shot noise dominates:

$$\frac{SNR_{Amp}}{SNR_{noAmp}} = \frac{1}{2n_{sp} + \frac{1}{G}} \approx \frac{1}{2n_{sp}}$$

- SNR is decreased by the optical amplification
- Best case: the spontaneous emission factor is 1 (correspond to completely inverted medium, $N_I = 0$)
- The SNR is at best decreased by a factor 2 (3 dB)

Noise figure of an optical amplifier

The noise figure (NF) is defined by the following expression

$$NF \equiv \frac{(SNR)_{in}}{(SNR)_{out}}$$

The SNR values are expressed as if measured with an ideal receiver placed before the amplifier (i.e. measuring input SNR) and after the amplifier (i.e. measuring the output SNR).

- Shot noise limited receiver
- 100 % quantum efficiency

Following the previous analysis, the minimum noise figure is

$$NF \approx 2n_{sp} \geq 2$$

Noise figure

For an EDFA the noise figure depends on the population inversion

N_1 and N_2 change along the length of the EDFA

- Pump power and signal power are not constant
- Rate equations can be solved numerically

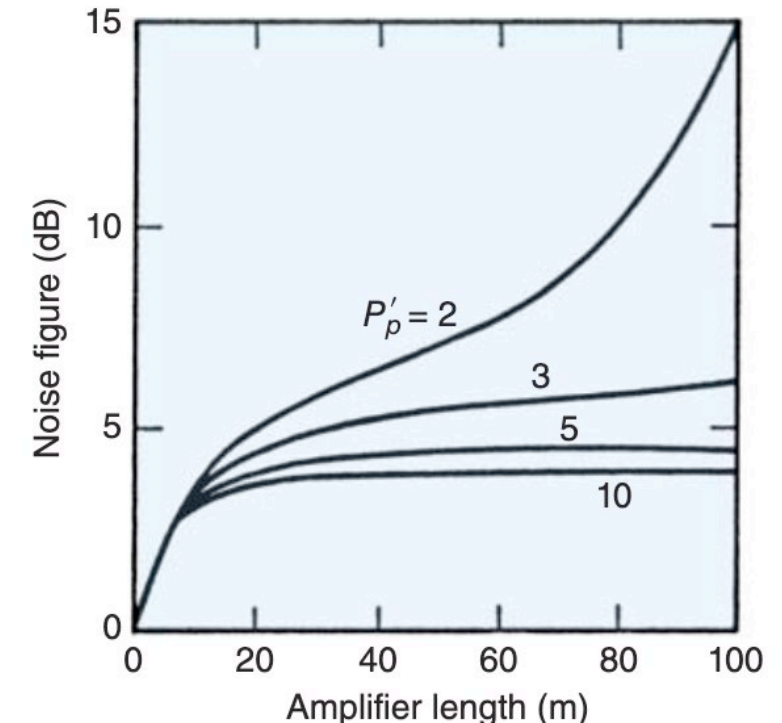
The noise figure is increased

- If population inversion is incomplete

Noise figure close to 3 dB (3.2 dB measured) are possible with 980 nm pumping

- No stimulated emission caused by the pump photons
- Much larger NF with 1480 nm pumping because the ground state will always be populated

Typical EDFA have noise figures around 4 to 6 dB



Optical amplifiers: comparison

Property	EDFA	Raman	SOA
Gain (dB)	40	15	20
Wavelength (nm)	1530-1625	1280-1650	1280-1650
Bandwidth at 3 dB (nm)	30-60	Pump dependent	60
Max saturation (dBm)	22	0.75 x pump	18
Polarization sensitivity	No	No	Yes
Noise figure	4-5	5	8
Pump power	25 dBm	> 30 dBm	< 400 mA
Size	Rack mounted, tabletop	Bulk module	Compact
Cost factor	Medium	High	Low